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Research article

ARI (p, d) Modelling and Forecasting of Nigeria's External Reserves

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Abstract

The need for proper knowledge of the behaviour of the Nigeria's external/foreign reserves cannot be over emphasized, as it would aid the Central Bank of Nigeria (CBN) and indeed the Federal government of Nigeria to make informed policies that would strengthen her economy. In the light of this, this paper attempts to identify and build a suitable and reliable Box-Jenkins model for modelling and forecasting the Nigeria's foreign reserves. Nigeria's 55 years foreign reserve data from January, 1960 to December, 2013 was used to perform analysis in *R*, were ARI (5, 1, 0) model with the smallest AIC and BIC statistics was found to out-perform the ARIMA (5, 1, 1) model. One year forecast was made with the best ARI (5, 1, 0) model and the Nigeria's foreign reserve was trending upwards. Although the point forecast values are higher than the observed values, interestingly the observed values are found to lie within the 95% confidence intervals. This paper therefore proposes the ARI (5, 1, 0) model for forecasting Nigeria's foreign reserves. **Copyright © WJPSR, all rights reserved.**

Keywords: Time Series**,** Box-Jenkins, ARIMA, Nigeria, Foreign Reserves

1.0 Introduction

External Reserve has widely been referred to as: international reserves, foreign reserves, or foreign exchange reserves. However, in this paper we have consistently used foreign reserves instead. A range of definitions of foreign reserve could be found in various learned Journals. The universally accepted definition was proposed by the IMF in its 5th edition of the Balance of Payment Manual as follows: *"consisting of official public sector foreign assets that are readily available to, and controlled by the monetary authorities, for direct financing of* *payment imbalances, and directly regulating the magnitude of such imbalances, through intervention in the exchange markets to affect the currency exchange rate and/or for other purposes".* The Central Bank of Nigeria (CBN) amended act No. 24 of 1999 [1], vested the CBN with the responsibility of monitoring and managing the country's Foreign Reserves. Virtually, every country of the world for one reason or the other holds foreign reserves. For example [2], Nigeria holds her's to: (i) Support and maintain confidence in the monetary policy and exchange rate management, including ability to interfere in the foreign exchange market (WDAS) to sustain the Naira, (ii) Service liabilities and debt of foreign currencies, (iii) Finance government expenditure abroad for example the importation of goods and services, (iv) Defence against emergencies or disaster, and (v) Offer a source of income, as the reserves can be invested in revenue generating instruments like the treasury bills, bonds, etc.

Time series analysis, particularly the Box-Jenkins (ARIMA) modelling of the Nigeria's foreign reserves has appeared in various scholarly articles. For example, Iwueze *et al.* [3] recommended the Auto Regressive Integrated Moving Average (ARIMA) process of order (2, 1, 0) for forecasting the natural log-transformed Nigeria's foreign reserves, using 11 years data (from January, 1999 to December, 2008), where the Nigeria's foreign reserves was found to be on the increase. Although, the point forecast from this model shows a large discrepancy from the observed and was attributed to the decline in income from petroleum products which is the major composition of the Nigeria's foreign reserves. However, the ARIMA (2, 1, 0) in [3] though a candidate model could not have been the best, instead the ARIMA (2, 1, 2) would have been preferred because the ACF plot of the first ordinary differenced log-transformed series showed a pronounced spike at lag 2 and cut-off thereafter. Zubair *et al.* [4] tentatively identified ARIMA (1, 2, 2) model as a suitable model for modelling and forecasting Nigeria's foreign reserves using a monthly 50 years data (January, 1960-December, 2008). The Nigeria's foreign reserves was found to be on the increase the paper further call on the Nigeria government to exercise fairness, justice, and equity for all in order to strengthen her economy. The proposed model in [4] was fitted with over differenced log-transformed series which could affect the forecasting power of the ARIMA (1, 2, 2) model, however, the ARIMA (1, 1, 2) model would have been preferred since a single ordinary difference would have rendered the series stationary, again, the model was chosen on the basis of the highest stationary R^2 statistics and the smallest information criterion statistics (BIC), it could be seen from the tentatively identified models in Table 3 that the ARIMA (1, 2, 1) model has a smaller stationary R^2, smaller (BIC) statistics of 0.639 and 14.347 respectively in contradiction with the ARIMA (1, 2, 2) model with corresponding statistics of 0.643 and 14.349, respectively.

Further application of the Box-Jenkins (ARIMA) time series modelling techniques is almost surely in literature for example, Akpanta *et al.* [5] applied the Box-Jenkins technique in modelling and forecasting the Nigeria's crude oil price using a monthly data from the CBN official website for the period of 33 years. Two ARIMA models: ARIMA (6, 1, 7), ARIMA (2, 1, 2) were tentatively entertained and the reduced ARIMA (2, 1, 2) was found to provide a best fit to the data and was used for forecast. Interestingly, the observed Nigeria's crude oil price does not show any statistically significant difference from the forecast values the paper concludes. Etuk *et al.* [6] identified and established the adequacy of the Seasonal ARIMA $(5,1,0)(0,1,1)_{12}$ for modelling and forecasting the amount of monthly rainfall in Portharcourt, Nigeria. Akpanta *et al.* [7] upheld and modelled the frequency of monthly rainfall in Umuahia the Abia State capital in NIgeria with SARIMA $(0, 0, 0)(0, 1, 1)_{12}$ model after favourably competing with SARIMA $(0, 0, 0)(1, 1, 1)_{12}$ model. However, the model was found satisfactory in forecasting the future Frequency of monthly rainfall in Umuahia and was therefore recommended for this task. Also, Akpanta *et al.* [8] found the parsimonious SARIMA $(1, 2, 1)(0, 0, 1)_{12}$ model after favourably competing with SARIMA $(11, 2, 1)(0, 0, 1)_{12}$ model as the best fit to the Nigeria's Consumers Price Index (CPI) data using 19 years monthly data from January, 1996 to December, 2013 constituting 216 observations. The SARIMA $(1, 2, 1)(0, 0, 1)_{12}$ model satisfactorily forecast the future Nigeria's CPI. The Nigeria's CPI was found to be on the increase and has neither fallen below 21.19 nor risen above 152.29 from January, 1996 to December, 2013 was one of the major findings among others, the paper unfolds.

This paper therefore attempts to identify and construct a more reliable Box-Jenkins ARIMA (p, d, q) model that would best explain the underlying generating process and satisfactorily forecast into the future of the Nigeria's foreign reserve.

2.0 Methods

The data used in this article which is a monthly sequential record of the Nigeria's foreign reserves constitutes a time series data. The Box-Jenkins, Auto Regressive Integrated Moving Average (ARIMA) time series modelling technique for stationary time series has been adopted for the analysis. Extensive literatures on the Box-Jenkins techniques and modelling procedures could be found in [9] and [10].

3.0 Materials and Analysis

The monthly time series of the Nigeria's foreign reserves data, used in this work was obtained from the official website of the Central Bank of Nigeria (CBN) [11], for the period of 55 years (January, 1960 - December, 2013). Analysis has extensively been performed using *R* programme for Windows.

More often than not, financial data are skewed and heavy tailed (not normally distributed). The data under study was investigated for normality and suitable variance-stabilizing transformation technique (the Box-Cox transformation) was used. Details of Box-Cox transformation could be found in [12], [13], [14], [15], [16], and [17]. This is basically, to ensure that the data is normally distributed and stationary if it is not. So, that the ARIMA modelling is reasonable. However, it is worth noting that results from such transformation would be difficult to interpret hence, a need to re-convert the results obtained from such transformed data to the original units and scale for easy interpretation. The data under study is skewed to the right with Skewness = 1.896865 and also heavy tailed with Kurtosis = 2.327915. Hence, the data does not follow the normal distribution, rather a skewed and heavy tailed distribution like the exponential, Pareto, Gumbel, Fréchet, etc., could explain the Nigeria's foreign reserve data better.

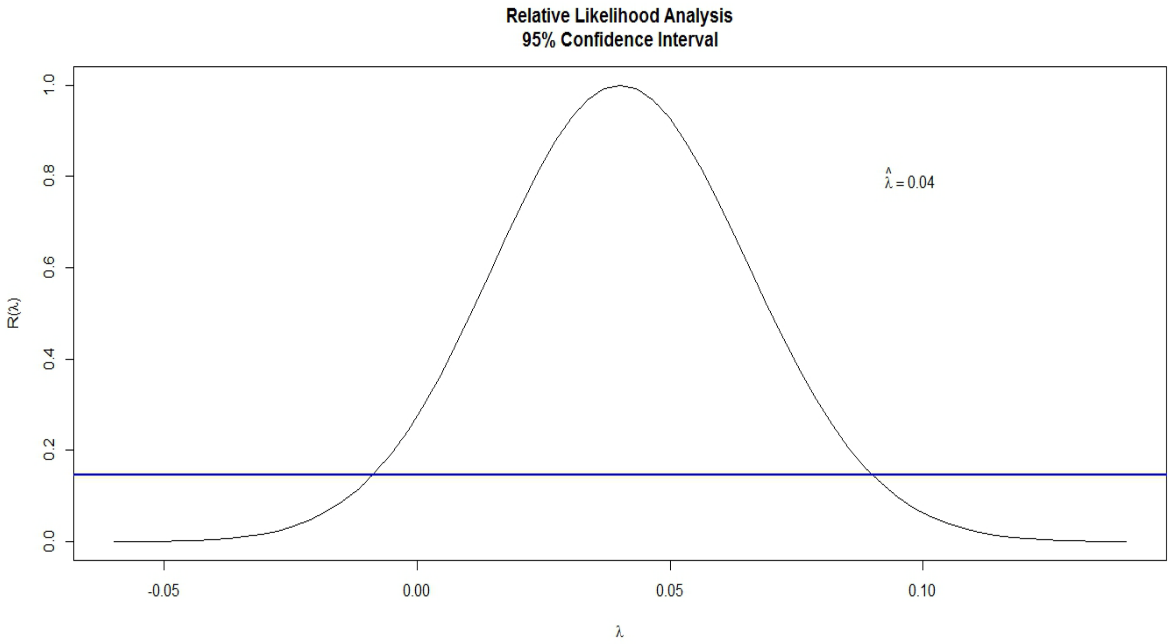

Figure 1: Box-Cox Transformation Indication Plot

The parameter $\hat{\lambda} = 0.04(\approx 0)$ is an indication that the data may require a natural log transformation to stabilize its variance and render it normally distributed.

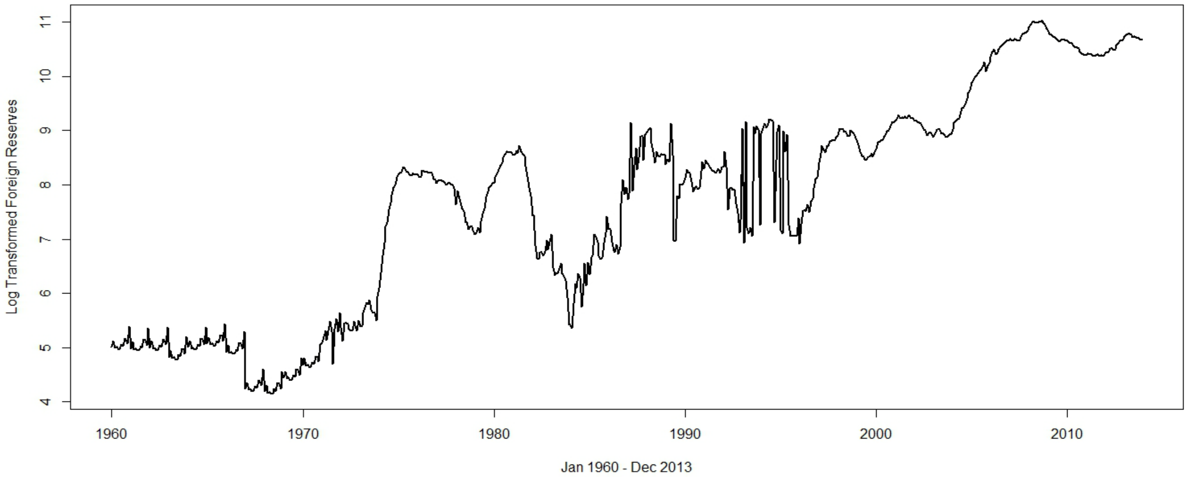

Figure 2: Log-Transformed Series

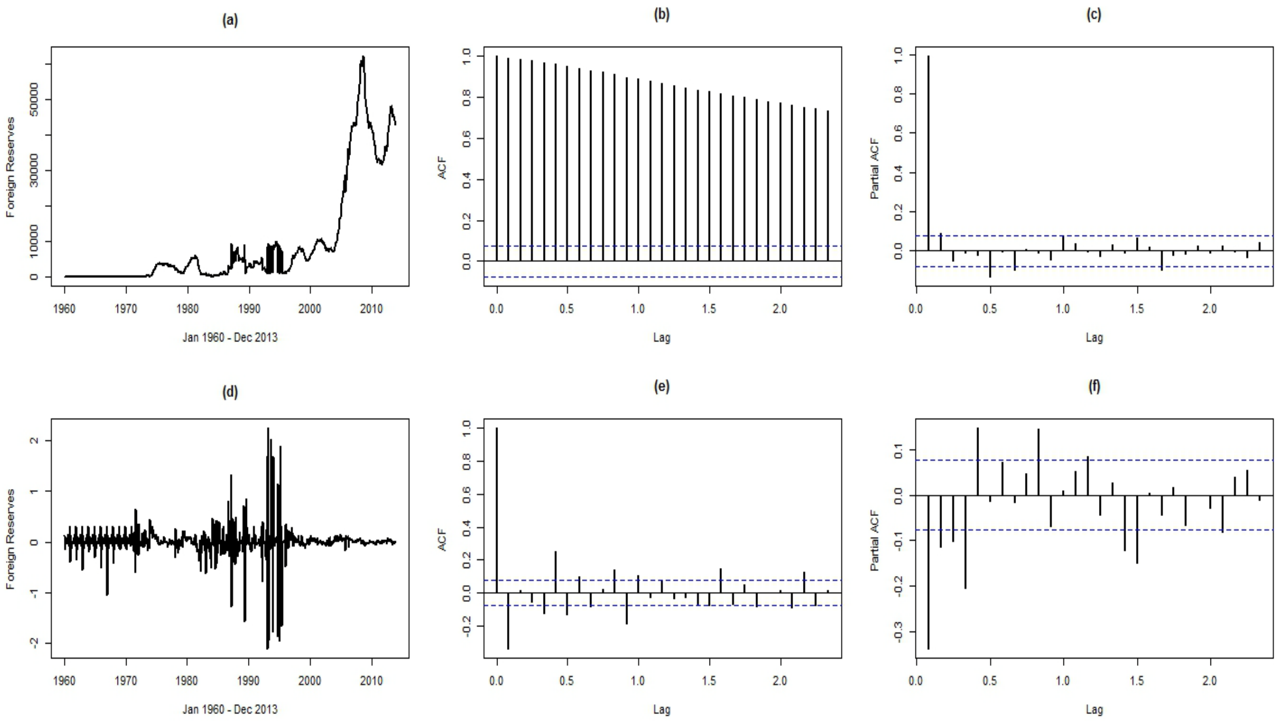

Figure 3:(a)-Time Series Plot of the Original data, (b) & (c)-ACF and PACF Plots Respectively of the Original data, (d)-Time Series Plot of the Differenced Logged-data, (e) & (f)-ACF and PACF Plots Respectively of the Differenced Logged-data.

Fig. (3a) shows the plot of the original time series, from which a slight linear trend could be seen suggesting non stationarity in the mean level while the variance appears to be relatively stable contradicting the Box-Cox suggestion above. However, in the sequel analysis we have ignored the suggestion of the subjective visual display of possible constant variance and upholding that of the Box-Cox log-transformation. Fig.(3b) and Fig.(3c) respectively depicts the ACF and the PACF of the original series. The ACF plot shows a very slow exponential decay and the PACF shows a cut-off immediately after lag 0, these are indications of a possible stochastic trend (non stationarity in the mean level) in the series. This is in agreement with Fig. (3a). Fig.(3d) shows that the first ordinary difference (i.e. $d = 1$) was enough to coerce the non stationary (non constant mean) log-transformed series to a stationary one as no obvious trend is seen. Figures (3e) and (3f) respectively depicts the ACF and PACF of the differenced log-transformed series, where Fig.(3e) shows a cut-off at lag 1 (indicating MA (1), i.e. $q = 1$) and Fig.(3f) shows a cut-off at lag 5 (indicating AR (5), i.e. $p = 5$), however, suggesting that the ARIMA (5, 1, 1) should be tentatively considered as a candidate model.

Table 1 Fitted ARIMA (5, 1, 1) Model

Parameters	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5	θ_1
Estimates	-0.6956	-0.2825	-0.2083	-0.1979	0.0806	0.3259
Se	0.2815	0.1240	0.0705	0.0722	0.0799	0.2823
AIC = 227.58 and **BIC** = 258.89						

All the parameters of the fitted ARIMA (5, 1, 1) model are statistically significant except θ_1 whose $z-value$ is 1.154446 which is less than the 5% upper tail critical value of 1.96. Dropping θ_1 from the fitted model, results to fitting a reduced and more parsimonious ARI (5, 1, 0) model as shown in Table 2.

Table 2: Fitted ARI (5, 1, 0) Model

Parameters	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5
Estimates	-0.3762	-0.1520	-0.1503	-0.1391	0.1501
Se	0.0388	0.0412	0.0411	0.0411	0.0387
AIC = 226.06 and **BIC** = 252.89					

All the parameters of the fitted ARI (5, 1, 0) model in Table 2 are statistically significant at $\alpha = 5\%$. Because, the estimated parameters are more than two times larger than their corresponding standard errors hence, having absolute $z-value$ above 1.96 (5% upper tail critical value). The diagnostic check of the fitted ARI (5, 1, 0) model is presented in Figure 4.

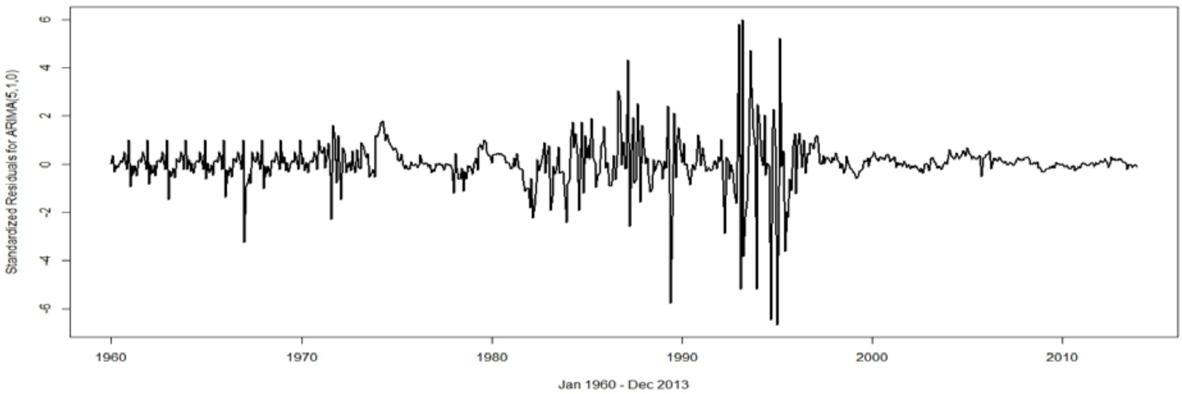

Figure 4: The Standardized Residual Plot of the Fitted ARI (5, 1, 0)

Fig. 4 shows the time series of the standardized residuals. The plot depicts no pattern, a clear indication that the standardized residuals behaves like a typical white noise process as required. The goodness of fit of the two tentatively identified models would be judged on the basis of their information criterion statistics (AIC[1] and

[1] Akaike Information Criterion, [18]

BIC[2]). The ARI (5, 1, 0) model with smaller AIC and BIC statistics is preferred over the ARIMA (5, 1, 1) model. Hence, is recommended for forecasting the future Nigeria's foreign reserve.

Let x_t denote the original series of the Nigeria's foreign reserve then, $y_t = \log_e(x_t)$ is the natural log-transformed series with which the ARI (5, 1, 0) model is fitted. The ARI (5, 1, 0) is algebraically written as:

Let the first ordinary differenced log-transformed series y_t be

$$w_t = (1 - B)y_t \quad (1)$$

then

$$w_t = \hat{\phi}_1 w_{t-1} + \hat{\phi}_2 w_{t-2} + \hat{\phi}_3 w_{t-3} + \hat{\phi}_4 w_{t-4} + \hat{\phi}_5 w_{t-5} + e_t \quad (2)$$

(2) is the fitted ARI (5, 1, 0) model and e_t is the error term which is an IID white noise process i.e., $e_t \sim IID(0, \sigma^2)$

Table 3 shows the twelve months forecast values of the log-transformed Nigeria's foreign reserves and their corresponding 95% confidence intervals.

Table 3: One Year (2014) Forecast Values of the Log Transformed Series

Log Transformed Series												
Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
UCL	11.235	11.331	11.419	11.472	11.513	11.605	11.660	11.720	11.769	11.814	11.868	11.912
Point Est.	10.675	10.672	10.676	10.673	10.671	10.673	10.672	10.673	10.672	10.672	10.673	10.672
LCL	10.116	10.012	9.933	9.874	9.828	9.742	9.68	9.626	9.576	9.530	9.478	9.432

Instead of using the naive converter

$$\hat{x}_{t(l)} = e^{\hat{y}_t(l)} \quad (3)$$

to return the log-transformed forecast series to its original scale, we use

$$\hat{x}_{t(l)} = e^{\hat{y}_{t(l)} + \frac{1}{2}\sigma^2_{e_{t(l)}}} \quad (4)$$

where $\sigma^2_{e_{t(l)}}$ is the variance of the l −step ahead forecast error

$$e_{t(l)} = y_{t+l} - \hat{y}_{t(l)}. \quad (5)$$

Hence, we present the twelve months forecast values of the Nigeria's foreign reserves in the original scale and their corresponding 95% confidence intervals in Table 4.

Table 4: One Year (2014) Forecast Values of the Original Series

Original Series												
Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
UCL	75728	83366	91063	95966	99992	109647	115870	123059	129202	135166	142607	149097
Point Est.	45083	45627	46536	46904	47235	48353	48973	4981	50464	51118	51982	52704
LCL	24739	22301	20596	19415	18550	17010	16052	15161	14412	13766	13065	12485

[2] Bayes Information Criterion, [19]

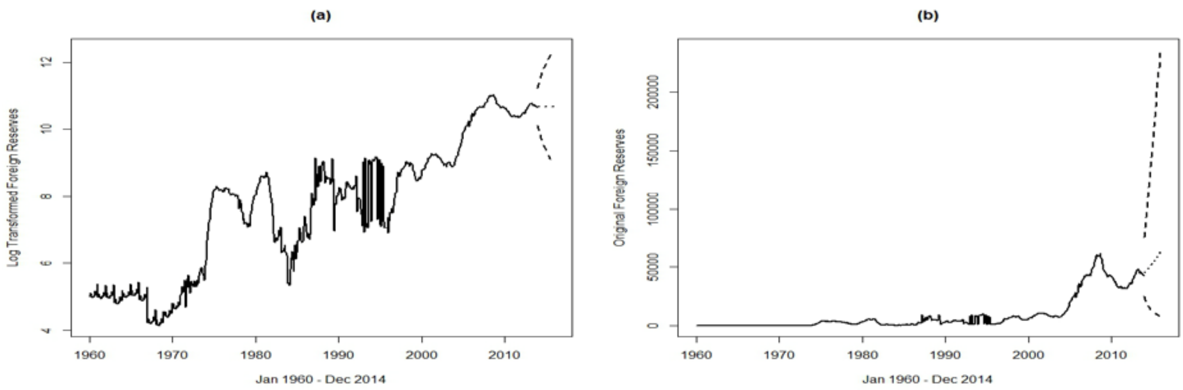

Figure 5: One Year Forecast of Nigeria's Foreign Reserve

Fig.(5a) shows the past (heavy lines) and future (dotted lines) time series of the Nigeria's foreign reserves on the natural logarithmic scale (log-transformed series) while, Fig. (5b) depicts the past (heavy lines) and future (dotted lines) time series of the Nigeria's foreign reserves on the original scale. The dashed lines in both plots indicate the 95% confidence bands. To investigate how satisfactorily the fitted ARI (5, 1, 0) model has performed in forecasting the future we present the plots of the observed time series and forecast values in a single graph as shown in Fig 6, using seven months data from January, 2014 to July, 2014.

Table 5: Observed Log-Transformed and Original Series from January, 2014 to July, 2014

Month	Jan	Feb	Mar	Apr	May	Jun	Jul
Logged	10.61319	10.51661	10.52941	10.52151	10.47441	10.52755	10.57299
Original	40667.56	36923.61	37399.22	37105.27	35398.10	37330.03	39065.42

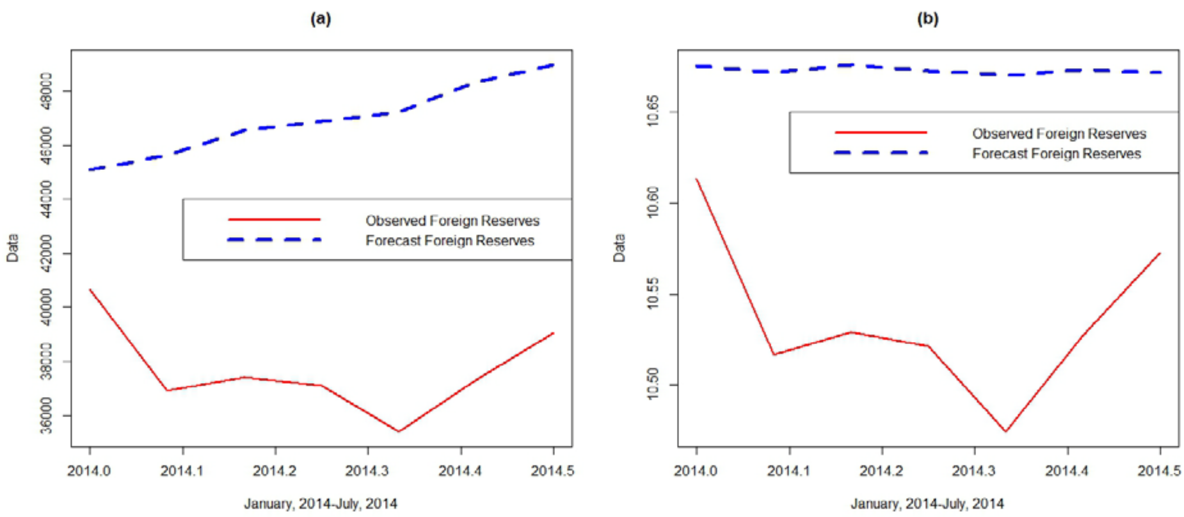

Figure 6: (a) Time Series Plot of the Original Series and Point Forecast Original Series of the Nigeria's Foreign Reserve Data. (b) Time Series Plot of the Log-Transformed Observed Series and Point Forecast Log-Transformed Series of the Nigeria's Foreign Reserve Data.

4.0 Discussion of Results and Conclusion

This paper has adopted the Box-Jenkins three step iterative model building procedure of identification, estimation, and diagnostic checking to model and forecast the Nigeria's foreign reserve using the monthly data spanning from January, 1960 to December, 2013. Two models ARIMA (5, 1, 1) and ARI (5, 1, 0) were tentatively entertained. The ARI (5, 1, 0) model was found to provide the best-fit to the foreign reserve data, because of its small information criteria statistics (AIC and BIC) in comparison to the ARIMA (5, 1, 1) this implies that the differenced log transformed series at time *t* depends on five (5) past values (*t-1, t-2, t-3, t-4,* and *t-5*) of the process. The moving average parameter in the ARIMA (5, 1, 1) model was found statistically insignificant and this led to the reduced and parsimonious ARI model. Twelve months forecast was made using the best fitting ARI (5, 1, 0) model and the forecast values were trending upwards. The point forecast values were all within the 95% confidence limits, but not consistent with the observed values (January, 2014 - July, 2014) as could be seen in Fig 6. This disagreement between the point forecast (large values) and the observed (small values) could be as a result of possible alteration in the underlying generating process which could be a consequence of the low revenue generation from the crude oil products since Nigeria is an oil dependent economy. This decrease in the Nigeria's foreign reserve raises a lot of questions begging for answers.

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